INFO 2950 JAN 29 2019

# Turning probabilities into other probabilities

$\neg C$ ← "not"

| | $C$ | $\neg C$ |
|---|---|---|
| $M$ | \|\|\|\| | 卌 \|\| 卌 |
| $\neg M$ | \|\| | 卌 卌 卌 卌 |

if this is missing, no way to recover it

$$P(C,M) = 4/38 = P(M,C)$$

we can always turn non-negative numbers into probabilities by normalizing ie divide by the sum of the numbers

$$P(C) = P(M,C) + P(\neg M, C)$$

| | $C$ | $\neg C$ | |
|---|---|---|---|
| $M$ | $4/38$ | $12/38$ | $16/38$ |
| $\neg M$ | $2/38$ | $20/38$ | $22/38$ |
| | $6/38$ | $32/38$ | |

if this is missing, we can recalculate it from the other 3 values $\rightarrow 1 - P(C,M) - P(C,\neg M) - P(\neg C, \neg M)$

"marginal" probability in the bottom margin

| | $C$ | $\neg C$ | |
|---|---|---|---|
| $M$ | $4/16$ | $12/16$ | $16/38$ |
| $\neg M$ | $2/22$ | $20/22$ | $22/38$ |

$$P(C \mid M) = \frac{P(C,M)}{P(M)} = \frac{\frac{4}{38}}{\frac{16}{38}} = 4/16 = 1/4$$

Oops! I left the original # of students in the denominator, so it looked like we could recover the original frequencies. But this has the same $P(C \mid M)$:

| | $C$ | $\neg C$ |
|---|---|---|
| $M$ | \| | \|\|\| |
| $\neg M$ | \|\| | 卌 卌 卌 卌 |

$$\frac{P(C,M)}{P(M)} = P(C \mid M)$$

$$P(C,M) = P(C \mid M)\,P(M)$$

$$P(M \mid C)\,P(C) = P(C \mid M)\,P(M)$$

$$P(M \mid C) = \frac{P(C \mid M)\,P(M)}{P(C)}$$

Bayes' Rule!!!

if B rolls 3 1/4

if G rolls 3 1/6

10% P(B)

| | 3 | ¬3 | |
|---|---|---|---|
| B | 1/4 | 3/4 | .1 |
| G | 1/6 | 5/6 | .9 |

| | 3 | ¬3 |
|---|---|---|
| B | $\frac{1}{4}\frac{1}{10}$ | $\frac{3}{4}\frac{1}{10}$ |
| G | $\frac{1}{6}\frac{9}{10}$ | $\frac{5}{6}\frac{9}{10}$ |

$$P(B|3) = \frac{P(3|B)P(B)}{P(3)}$$

$$= \frac{\frac{1}{4} \cdot \frac{1}{10}}{\frac{1}{4}\frac{1}{10} + \frac{1}{6}\frac{9}{10}}$$

Notice that this would be the numerator for P(G|3)!

$$= \frac{\frac{1}{4}}{\frac{1}{4} + \frac{9}{6}} = .1429$$